Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2020

Subject Name: Mathematical Methods - I

Subject Code: 5SC03MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 27/02/2020** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a) Write Euler's formula for Fourier coefficient of $f(x)$. (02)

b) Define: Finite Fourier sine transform (02)

c) State Dirichlet's condition. (02)

d) Complete the formula: $F_s\{f(x)\cos ax\} =$ ____________. (01)

Q-2 **Attempt all questions** **(14)**

a) Find the Fourier series of $f(x) = x + x^2; -\pi < x < \pi$. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6$. (07)

b) Prove that $f(x) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$, (04)

where $f(x) = \begin{cases} -k & , -\pi < x < 0 \\ k & , 0 < x < \pi \end{cases}$

c) State and prove Parseval's Formula. (03)

OR

Q-2 **Attempt all questions** **(14)**

a) Find a Fourier series of $f(x) = 1 + \sin x; -1 < x < 1$. (05)

b) Obtain the Fourier expansion of $f(x) = \sinh x$ in $(-\pi, \pi)$. (05)

c) Using the Fourier series expansion of $f(x) = \frac{(\pi - x)^2}{4}$ in $(0, 2\pi)$, and hence deduce that $f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$. (04)

Q-3		**Attempt all questions**	**(14)**
	a)	State and prove Fourier integral theorem.	(05)
	b)	Solve: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ given that $u_x(0,t)=u_x(6,t)=0$ and $u(x,0)=2x$, where $0<x<6$ and $t>0$	(05)
	c)	Solve the Fourier transform of $f(x)=\begin{cases} a-\lvert x\rvert, if \lvert x\rvert<a \\ 0 \quad , if \lvert x\rvert>a \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$ and $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$.	(04)

OR

Q-3		**Attempt all questions**	**(14)**
	a)	Solve the heat conduction problem described by PDE: $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$, $0<x<\infty$, $t>0$ BC : $u(0,t)=0$, $t>0$ IC: $u(x,0)=f(x), 0<x<\infty$, and $u, \frac{\partial u}{\partial x}$ both tend to zero as $x\to\infty$, by using Fourier sine transform.	(07)
	b)	Express $f(x)=\begin{cases} e^{kx} ; x<0 \\ e^{-kx} ; x<0 \end{cases}$ as a Fourier integral and show that $\frac{\pi}{2k}e^{-kx}=\frac{2}{\pi}\int_0^\infty \frac{\cos\lambda x}{\lambda^2+k^2}\,d\lambda,\ x>0, k>0.$	(04)
	c)	Find the Fourier cosine and sine transform of xe^{-ax}.	(03)

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a)	Find: $L^{-1}\left(\frac{1}{(s^2+4)^2}\right)$	(02)
	b)	Evaluate: $\int_0^\infty e^{-st}\left(4+\cos^2 2\theta\right)dt+\int_0^\infty e^{-st}\sin^2 2\theta\, dt$	(02)
	c)	Find the Z-transform of $\cos n\theta$.	(02)
	d)	State Final value theorem for Z-transform.	(01)
Q-5		**Attempt all questions**	**(14)**
	a)	Find: $L\left(t\left(\frac{\cos t}{e^t}\right)^2\right)$	(04)
	b)	Evaluate: $\int_0^\infty e^{-t}\int_0^t \frac{\sin u}{u}\,du\,dt$	(03)

c) Using convolution theorem evaluate : (04)

i. $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$.

ii. $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$.

d) State and prove first shifting theorem. (03)

OR

Q-5 **Attempt all questions** **(14)**

a) Solve $\frac{dx}{dt}+\frac{dy}{dt}+x-y=e^{-t}$, $\frac{dx}{dt}+\frac{dy}{dt}+2x+y=e^{t}$ by using Laplace transforms. (07)

Where $x(0)=1, y(0)=0$.

b) Find: $L\left(t^2 \sin at\right)$ (04)

c) If $f(t)=\begin{cases}3, & 0\le t\le 5\\ 0, & t>5\end{cases}$ then find $L\left(f'(t)\right)$. (03)

Q-6 **Attempt all questions** **(14)**

a) Write Rodrigues formula, generating function for Hermite polynomials and also first three Hermite polynomials. (05)

b) Find Z-transform for the following: (05)

i) $a^n \sin n\theta$ ii) $n^2 e^{an}$

c) Find $Z^{-1}\left(\frac{2z(2z-1)}{z^3-5z^2+8z-4}\right)$. (04)

OR

Q-6 **Attempt all Questions** **(14)**

a) Solve $\frac{dy}{dt}+2y+\int_0^t y(t)\,dt=\sin t; y(0)=1$ by using Laplace transforms. (06)

b) State and prove multiplication by n rule for Z-transform and also write its generalized form. (04)

c) State and prove convolution theorem for Z-transform and evaluate $Z^{-1}\left(\frac{z^2}{(z-2)^2}\right)$. (04)